

Equation 2 is still three dimensional with three independent diffusion constants, D_1 , D_2 and D_3 corresponding to the three crystalline axes. However, if the material has cubic symmetry, it can be shown that the bulk diffusion is isotropic; that is, independent of crystalline direction with $D_1=D_2=D_3=D$ where D is called the diffusion constant. If the geometry is such that the concentration is a function only of a single cartesian coordinate and time, equation 2 reduces to the usual one dimensional diffusion equation:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \quad (3)$$

Solutions of equation 3 for various initial conditions are tabulated and discussed by Crank.¹⁴ The solution for a semi-infinite rod with a delta-function source on the end is given by equation 4:

$$C = \frac{\alpha}{2\sqrt{\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right) \quad (4)$$

where α is a constant associated with the strength of the source, t is the anneal time at constant pressure and temperature, and x is the penetration distance measured from the source. A rule of thumb for determining the length of rod needed in order for equation 4 to be a good approximation for the solution of the diffusion equation for a finite rod is $l > 2\sqrt{Dt}$.

If we take the log of both sides of equation 4,

$$\ln C = \frac{-x^2}{4Dt} + \ln\left(\frac{\alpha}{2\sqrt{\pi Dt}}\right) \quad (5)$$

we see that $-1/4Dt$ is the slope of the log of the concentration vs the square of the penetration distance curve.